Algorithm specification for the Magnetic Diagnostics onboard LPF

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Acronyms

ASD	Astrium GmbH, Astrium Deutschland/Germany
BLR	Bottom-Left-Rear
CoM	Centre of Mass
CI	Configuration Item
DC	Direct Current
DDS	Data Management and Diagnostics Sub-System
DFACS	Drag Free and Attitude Control System
DMU	Data Management Unit
ESA	European Space Agency
ESAC	European Space Operations Centre (Darmstadt)
GW	Gravitational Wave
IEEC	Institut d'Estudis Espacials de Catalunya
LISA	Laser Interferometer Space Antenna
LPF	LISA Pathfinder (formerly SMART-2)
LTP	LISA Technology Package
LTPDA	LTP Data Analysis
MBW	Measurement Bandwidth
STOC	Science and Technology Operations Centre
TM	Test mass
VE	Vacuum Enclosure

1 Applicable documents

Ref.	Title	Doc Number	Issue	Date
AD1	Science Requirements and Top-level Architecture Definition for the LISA Technology Package (LTP) on Board LISA Pathfinder (SMART-2)	LTPA-UTN-ScRD	3.1	30/Jun/2005
AD2	DDS Subsystem Specification	S2-ASD-RS-3004	4.4	02/May/2007

2 Reference documents

Ref.	Title	Doc Number	Issue	Date
RD1	DFACS User Manual	S2-ASD-MA-2004	1.0	22/May/2008
RD2	Measurement of LTP dynamical coefficients by system identification	S2-UTN-TN-3045	1.1	05/Jun/2007
RD3	Diagnostic elements-DAUs interface configuration	S2-IEC-TN-3023	1.3	03/Dec/07
RD4	DDS Design Specification	S2-NTE-DS-3001	1.3	29/Sep/2006
RD5	J Schoukens and R Pintelon, *Identification of Linear Systems*	—	—	Pergamon Press, 1991
RD6	A Schnabel, L Trougnou and E Balaguer, *LTP EM TM Remnant Magnetic Moment Measurements*	S2-EST-TR-3002	2.0	2/Feb/2007
RD7	M Nofrarias and J Sanjuán, *Thermal experiments on board LPF*	S2-IEC-TN-3042	0.4	12/Sep/2008
RD8	A Schleicher, N Brandt and T Ziegler, *DFACS External ICD*	S2-ASD-ICD-2011	1.4	15/Sep/2008
RD9	J Fauste, M Armano, *LTP Investigations Description Document*	S2-ESAC-TN-5005	1.0	20/Oct/2008
RD10	Paul McNamara, *DDS algorithm user requirement document*			21/Jul/2009
RD11	Alberto Lobo, Ignacio Mateos, *Test report for LTP FM Magnetic Diagnostics*	S2-IEC-TR-3067	1.1	22/Oct/2008
RD12	M Diaz-Aguilo, Alberto Lobo, E Garcia-Berro, *Neural Network algorithms for magnetic diagnostics in the LTP*	S2-IEC-TN-3052	1.0	11/Mar/2009

3 Introduction

3.1 Scope

This Algorithm Specification Note specifies the algorithm to simulate and compute the dynamic effects (forces and torques) on both TM's of the LISA Technology Package (LTP) caused by the magnetic field generated by the two on-board coils of the Data and Diagnostics Subsystem (DDS).

3.2 Statement of the problem

The LTP is equipped with two magnetic coils, whose common axis is the line joining the centres of the two proof masses (TMs), and placed in the outer sides of each of the vacuum enclosures (VE). By feeding currents of various frequencies in the MBW to these coils one generates controlled magnetic fields, which in turn generate forces and torques on the TMs such that the latter move back and forth, and rotate, away from their centred position. The LTP readout channels detect the motion signals, thereby enabling us to estimate the magnetic properties of the TMs, i.e., the remanent magnetic moment and magnetic susceptibility of the TMs. Layout details are shown in Figures 3.1 and 3.2.

3.2.1 Magnetic forces and torques

A material body with some magnetic susceptibility and remanent magnetic moment submitted to the action of a magnetic induction field, feels a force and a torque which are given by

$$\mathbf{F} = \left\langle \left[\left(\mathbf{M} + \frac{\chi}{\mu_0}\mathbf{B} \right) \cdot \boldsymbol{\nabla} \right] \mathbf{B} \right\rangle V \tag{3.1}$$

and

$$\mathbf{N} = \left\langle \mathbf{M}\times\mathbf{B} + \mathbf{r}\times \left[(\mathbf{M}\cdot\boldsymbol{\nabla})\,\mathbf{B} + \frac{\chi}{\mu_0}\,(\mathbf{B}\cdot\boldsymbol{\nabla})\,\mathbf{B} \right] \right\rangle V \tag{3.2}$$

respectively. The meaning of symbols in the above formulas is as follows:

$\mathbf{B}$	Magnetic induction field in the TM
$\mathbf{M}$	Density of magnetic moment (*magnetisation*) of the TM
$\mathbf{r}$	Vector distance to the TM centre of mass
V	Volume of the TM
χ	Magnetic *susceptibility* of the TM
μ_0	Vacuum magnetic constant ($4\pi\times10^{-7}$ m kg s^{-2} A^{-2})

Finally, $\langle\cdots\rangle$ indicates TM volume avergage of enclosed quantity. For example, for any magnitude $f(\mathbf{x})$, such average is defined by the volume integral

$$\langle f\rangle \equiv \frac{1}{V}\int_V f(\mathbf{x})\,d^3x \tag{3.3}$$

over the TM volume.

We will calculate torques with respect to the body's Centre of Mass (CoM), and assume the magnetisation $\mathbf{M}$ is *uniform* across the TM. Under this assumption, the torque term $\left\langle \mathbf{r} \times \left[\frac{\chi}{\mu_0}\left(\mathbf{B}\cdot\boldsymbol{\nabla}\right)\mathbf{B}\right]\right\rangle V$ vanishes due to the axial symmetry around the x-axis of the field injected by the coil. Therefore equation (3.2) simplifies to

$$\mathbf{N} = \left\langle \mathbf{M} \times \mathbf{B} + \mathbf{r} \times \left[\left(\mathbf{M}\cdot\boldsymbol{\nabla}\right)\mathbf{B}\right]\right\rangle V \tag{3.4}$$

in this case.

In order to evaluate equations (3.1) and (3.4), we need the field created by the coils at the TM positions, and also the gradients of the components of $\mathbf{B}$, i.e., the matrix

$$\boldsymbol{\nabla}\mathbf{B} = \begin{pmatrix} \dfrac{\partial B_x}{\partial x} & \dfrac{\partial B_x}{\partial y} & \dfrac{\partial B_x}{\partial z} \\ \dfrac{\partial B_y}{\partial x} & \dfrac{\partial B_y}{\partial y} & \dfrac{\partial B_y}{\partial z} \\ \dfrac{\partial B_z}{\partial x} & \dfrac{\partial B_z}{\partial y} & \dfrac{\partial B_z}{\partial z} \end{pmatrix} \tag{3.5}$$

Note that the above is a symmetric and traceless matrix. This is because of the properties of the induction field

$$\boldsymbol{\nabla} \times \mathbf{B} = \boldsymbol{\nabla}\cdot\mathbf{B} = 0 \tag{3.6}$$

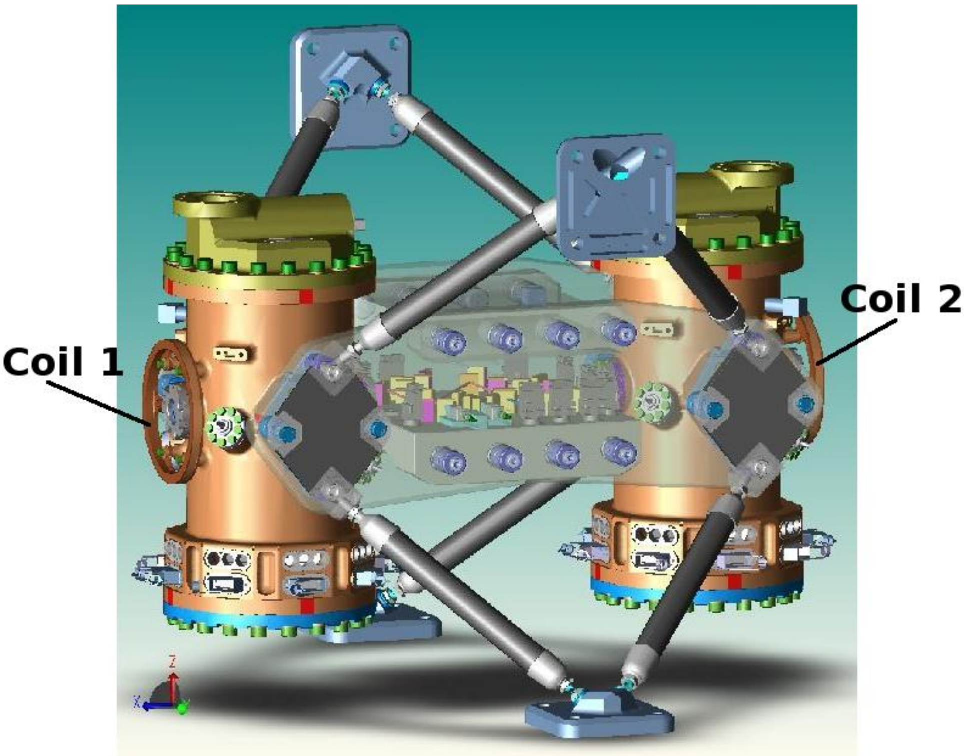


Figure 3.1: Design picture of the LTP. Coils can be seen around the VE flanges in the left and right ends of the drawing.

Analytic expressions can of course also be found for $\nabla\mathbf{B}$, but they are quite involved and not specially useful for the purposes of this note anyway.

The specific step by step procedure and guidelines for the calculations will be given in the Algorithm specification section below.

3.2.2 Hardware description

Details of the geometry of the hardware is necessary to perform the calculations. The layout of the experiment is illustrated in figure 3.2; the test mass positions are specified in table 3.1, and the coil positions in table 3.2. Other relevant geometry parameters are:

L	Test mass side:	46mm
a	Coil radius:	56.5mm
N	Number of coil turns:	2400

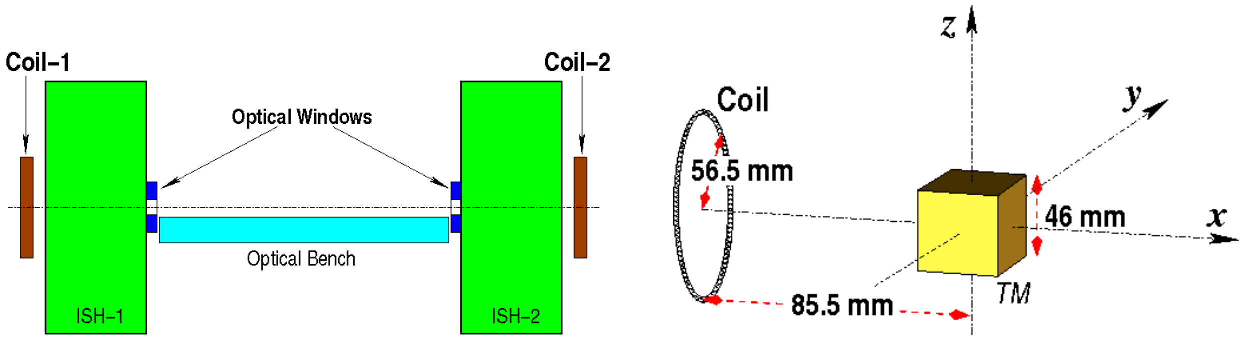


Figure 3.2: Global concept drawing of the coils' positions (left), and geometric data (right), applicable to either TM.

Table 3.1: Positions of the Test Masses referred to a coordinate system fixed to the spacecraft.

Test Mass	x [m]	y [m]	z [m]
1	−0.1880	0	0.4784
2	0.1880	0	0.4784

Table 3.2: Positions of the coils. They are referred to a coordinate system fixed to the spacecraft.

Coils	x [m]	y [m]	z [m]
1	−0.2735	0	0.4784
2	0.2735	0	0.4784

4 Algorithm Specification

The calculation of magnetic dynamics data (forces and torques) on the TM's will be split into four main steps —see figure 4.1.:

1. Computation of the magnetic field generated by the coils at the interior volume of each TM position, for a given current amplitude in the corresponding coil.
2. Computation of the force and torque amplitude due to the above magnetic field within each TM volume by solving the integrals implied in equations (3.1) and (3.4).
3. Computation of the current signals injected into the coils: commanded signal plus intensity of noise in their generation.
4. Computation of the forces and torques *time series* from the actual time varying current circulating in the coils.

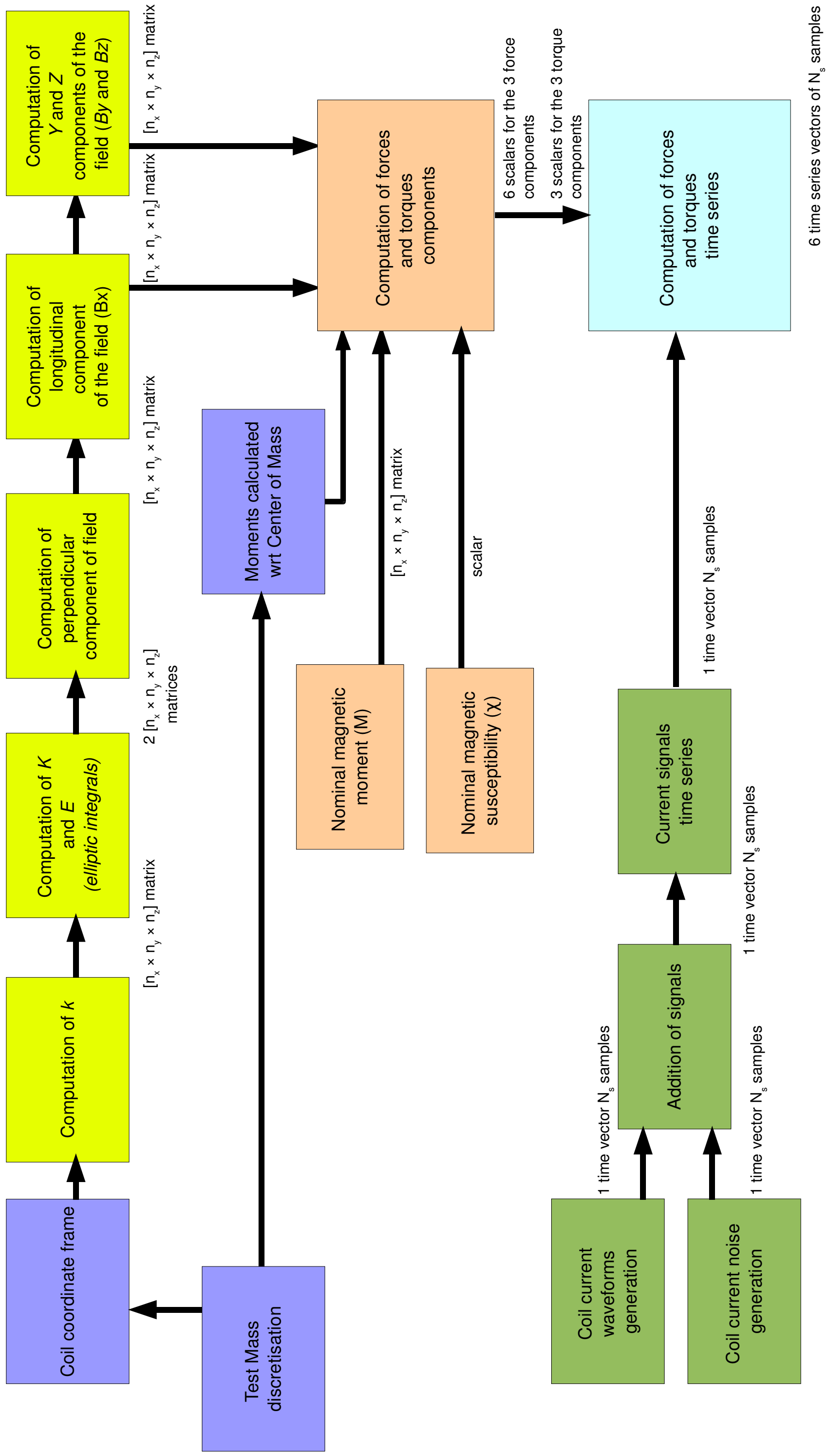


Figure 4.1: Computation flow diagram to obtain the forces and torques time series created by one coil on the closest test mass. In blue, we represent the discretisation and the necessary changes of coordinates; in yellow, the magnetic field related computations; in light orange the force and torque evaluation; in green the current signals computation; and in light blue, the final output of the algorithm.

4.1 Notation

Most of the calculations require evaluations of integrals. There are a wealth of mathematical methods and numerical routines in the literature and in the Internet which can be used as best suits a specific purpose. In the present context, however, all functions implied in the calculations are rather smooth and well behaved, so that basically all integrals can be estimated by finite Riemann sums, i.e., a grid of suitable finesse is defined in the integration domain, then the integrand is evaluated at a point in each grid cell, then multiplied by the cell's volume, and finally all this products are summed up. The convergence of this procedure to the true result can be assessed by trying successive refinements of the grid until no changes in the results are observed.

The integration domain in our case will be the TMs volume, which will be approximated by a cube of $L = 46$ millimetres to the side in each case, i.e., bevelled corners and notched faces to accommodate caging mechanism fingers and plungers will be (safely) disregarded. The grid elements will thus define parallelepipeds of side sizes $(L/n_x, L/n_y, L/n_z)$, where (n_x, n_y, n_z) are the number of grid points on each dimension.

This process results in space varying functions being discretely sampled and thereby converted into 3D matrices of dimension $[n_x \times n_y \times n_z]$. We shall use the following ***notation***:

1. Vector quantities in three dimensional space will be represented with **bold** symbols. For example, the magnetic field $\mathbf{B}$, the position of a point in space $\mathbf{x}$, etc.

2. Scalar quantities and components of vectors and matrices will be noted in *italics*. For example, the susceptibility χ, the components of the magnetic field B_x, B_y, B_z, etc.

3. Sampled quantities will be written with integer arguments in *square brackets*. For example, if a function $f(t)$ is sampled at times $t_1, \ldots, t_n$ then $f[i]$ will be the i-th sample, i.e., $f[i] = f(t_i)$, $i = 1, \ldots, n$.

4. The above also applies to 3D sampling in a grid like the one described above. For example, the grid vertexes themselves have coordinates

$$\begin{aligned} x[l_x] &= x_0 + l_x \Delta x \ , \quad 1 \leq l_x \leq n_x \\ y[l_y] &= y_0 + l_y \Delta y \ , \quad 1 \leq l_y \leq n_y \\ z[l_z] &= z_0 + l_z \Delta z \ , \quad 1 \leq l_z \leq n_z \end{aligned} \tag{4.1}$$

where (x_0, y_0, z_0) is the position of the left-bottom-rear (LBR) corner of the TM, and

$$\Delta x = \frac{L}{n_x} \ , \quad \Delta y = \frac{L}{n_y} \ , \quad \Delta z = \frac{L}{n_z} \tag{4.2}$$

A function $f(\mathbf{x})$ of the space coordinates sampled at the grid points will represented by

$$f(\mathbf{x}) \ \longrightarrow \ f[l_x, l_y, l_z] = f(x[l_x], y[l_y], z[l_z]) \tag{4.3}$$

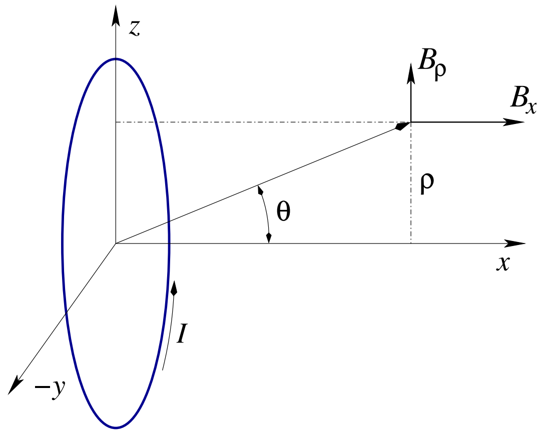
In the following we go through the various steps which lead to the final evaluation of forces and torques exerted on the TMs by the action of the LTP induction coils, as listed in section 4. They are shown in detail in figure 4.2, and expanded in the ensuing sections.

4.2 Algorithm overview

Step	Sub-step	Magnitude	Notation	Samples	Output
1	1.1	k-parameter	k	$k[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
	1.2	Elliptic integral 1st kind	$K(k)$	$K[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
		Elliptic integral 2nd kind	$E(k)$	$E[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
	1.3	Transverse mag. field	$B_\rho(\mathbf{x})$	$B_\rho[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
	1.4	Longitudinal mag. field	$B_x(\mathbf{x})$	$B_x[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
	1.5	Cartesian components	$B_y(\mathbf{x})$	$B_y[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
			$B_z(\mathbf{x})$	$B_z[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
	1.6	Gradient components	$\partial B_x/\partial x(\mathbf{x})$	$\partial B_x/\partial x[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
			$\partial B_x/\partial y(\mathbf{x})$	$\partial B_x/\partial y[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
			$\partial B_x/\partial z(\mathbf{x})$	$\partial B_x/\partial z[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
			$\partial B_y/\partial y(\mathbf{x})$	$\partial B_y/\partial y[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
			$\partial B_y/\partial z(\mathbf{x})$	$\partial B_y/\partial z[\boldsymbol{\ell}]$	matrix $n_x{\times}n_y{\times}n_z$
2	2.1	Longitudinal force, x	F_{x_1}, F_{x_2}	F_{x_1}, F_{x_2}	2 scalars
	2.2	Transverse force, y	F_{y_1}, F_{y_2}	F_{y_1}, F_{y_2}	2 scalars
	2.3	Transverse force, z	F_{z_1}, F_{z_2}	F_{z_1}, F_{z_2}	2 scalars
	2.4	Longitudinal torque, x	N_x	N_{x_1}	1 scalar
	2.5	Transverse torque, y	N_y	N_{y_1}	1 scalar
	2.6	Transverse torque, z	N_z	N_{z_1}	1 scalar
3	3.1	Electric current	$i_{\text{noise-free}}(t)$	$i_{\text{noise-free}}[\mathrm{K}]$	vector N_s
	3.2	Current noise	$n(t)$	$n[\mathrm{K}]$	vector N_s
	3.3	Actual current	$i(t)$	$i[\mathrm{K}]$	vector N_s
4	4.1	Force time series F_x	$F_x(t)$	$F_x[\mathrm{K}]$	vector N_s
		Force time series F_y	$F_y(t)$	$F_y[\mathrm{K}]$	vector N_s
		Force time series F_z	$F_z(t)$	$F_z[\mathrm{K}]$	vector N_s
	4.2	Torque time series N_x	$N_x(t)$	$N_x[\mathrm{K}]$	vector N_s
		Torque time series N_x	$N_y(t)$	$N_y[\mathrm{K}]$	vector N_s
		Torque time series N_x	$N_z(t)$	$N_z[\mathrm{K}]$	vector N_s

Figure 4.2: Step-by-step computation procedure.

4.3 Magnetic field calculation (Step 1)



When the magnetic induction field is created by an induction coil, the laws of Classical Magnetic Theory can be applied to obtain formulas which produce the values of the field and its gradient at any position in space. For slowly varying coil currents and short distances from them, radiative effects can be safely neglected.

The configuration of coils and TMs is that shown in Figure 3.2. The system has axial symmetry, hence only *parallel* (B_x) and *transverse* (B_ρ) components of the field $\mathbf{B}$ are different from zero —see figure beside. The magnetic field is calculated by means of Ampère's induction laws, assuming a coil of negligible thickness and a wire winding of N turns. The result involves elliptic integrals of the first and second kind ($K(k)$ and $E(k)$), which therefore need to be evaluated —see section below for full details.

4.3.1 Elliptic integrals (Step 1.1 and Step 1.2)

Once a value of the argument k is fixed, e.g., for a particular grid cell in the TM, the elliptic functions $K(k)$ and $E(k)$ are evaluated there. Elliptic integrals are well documented in abundant manuals and software literature. It is recommended that use be made of suitable subroutines to calculate them, basically on grounds of reliability.

The equations to compute are the following, and they should be evaluated in the specified order and for each of the grid points:

$$k^2 = \frac{4a\rho}{x^2 + (a+\rho)^2}\ , \qquad \rho^2 = y^2 + z^2 \tag{4.4}$$

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2\phi)^{-1/2}\, d\phi \tag{4.5}$$

$$E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2\phi)^{1/2}\, d\phi \tag{4.6}$$

4.3.2 Magnetic field components (Steps 1.3 through 1.5)

In order to evaluate the transverse and longitudinal components of the magnetic field, the following numerical values should be adopted:

$$\begin{aligned} N &= 2\,400 \text{ wire turns around each coil} \\ I &= 1 \text{ Ampere DC current for reference later on} \\ \mu_o/4\pi &= 10^{-7}\ \mathrm{m\,kg\,s^{-2}\,A^{-2}}, \text{ the magnetic constant} \end{aligned}$$

And here below the equation to compute the field components:

$$B_\rho(x,\rho) = \frac{\mu_0}{4\pi}\,\frac{\pi a^2 N\,I}{(a\rho^{3/2})}\,\frac{k}{\pi}\,\frac{x}{a}\,\left[-K(k) + \frac{1-k^2/2}{1-k^2}\,E(k)\right] \tag{4.7}$$

and

$$B_x(x,\rho) = \frac{\mu_0}{4\pi}\,\frac{\pi a^2 N\,I}{(a\rho^{3/2})}\,\frac{k}{\pi}\,\left[\frac{1}{2}\,\frac{k^2}{1-k^2}\,E(k)\right] - \frac{\rho}{x}\,B_\rho \tag{4.8}$$

Once the transverse field B_ρ is known, it takes two multiplications to resolve it into its Cartesian components B_y and B_z, as shown in equations (4.9) and (4.10). The magnetic field computation algorithm thus yields three $[n_x \times n_y \times n_z]$ 3-dimensional matrices per TM:

$$B_y = \frac{y}{\rho}\,B_\rho = \frac{y}{\sqrt{y^2+z^2}}\,B_\rho \tag{4.9}$$

and

$$B_z = \frac{z}{\rho}\,B_\rho = \frac{z}{\sqrt{y^2+z^2}}\,B_\rho \tag{4.10}$$

The output of these first steps are therefore:

- B_x: 3 dimensional matrix $[n_x \times n_y \times n_z]$ representing the B_x value in the TM.
- B_y: 3 dimensional matrix $[n_x \times n_y \times n_z]$ representing the B_y value in the TM.
- B_z: 3 dimensional matrix $[n_x \times n_y \times n_z]$ representing the B_z value in the TM.

4.3.3 Magnetic field gradient components (Step 1.6)

As already mentioned, to write down analytic expressions for the derivatives of the field is not very useful. It is instead recommended that such derivatives be approximated by finite difference between neighbouring points. For example,

$$\frac{\partial B_x}{\partial y}(x,y,z) \simeq \frac{B_x(x,y+\varepsilon,z) - B_x(x,y,z)}{\varepsilon} \tag{4.11}$$

and likewise for any other derivatives. It has to be noted that, while (x,y,z) should be a grid point, it is *not* required that $(x,y+\varepsilon,z)$ be also a grid point. Note however that:

- Having $(x,y+\varepsilon,z)$ in the grid is useful because the magnetic field in it has already been calculated in steps 1.2 through 1.5.
- Nevertheless, the criterion to select grid spacings is given by their ability to make the force and torque integrals converge —see below— and this may or may not coincide with the best choice to accurately estimate the field derivatives.

A possible alternative approach is to make use of more elaborate routines, available in software libraries, to evaluate the derivatives of the field.

Once a criterion has been adopted, the five independent partial derivatives enumerated in figure 4.2 can be calculated.

In this step, five 3-dimensional matrices for each of the independent gradient components result.

Table 4.1: On-axis magnetic field and longitudinal gradient values at three points on the TM for a 1 mA DC current fed to the 2 400 turn LTP coil. They are the closest point to the TM, the TM centre and the farthest point along the x-axis. Geometrical data are displayed in figure 3.2, right panel.

	$x = 62.5$ mm	$x = 85.5$ mm	$x = 108.5$ mm
B_x [μT]	8.05	4.47	2.63
$\partial B_x/\partial x$ [μT/m]	-212.6	-109.2	-57.2

4.3.4 Verification results for the magnetic field on the x-axis

In order to partly verify the correctness of the field calculations, a specific and simple configuration can be used. For example one can compute the on-axis field with the well known textbook formula

$$B_x(x, \rho = 0) = \frac{\mu_0}{4\pi} \frac{2\pi a^2 N I}{(a^2 + x^2)^{3/2}} \qquad \text{and} \qquad B_\rho(x, \rho = 0) = 0 \tag{4.12}$$

and this should reproduce the results obtained with equations (4.7) and (4.8), in the appropriate limit. An additional check is the x-gradient of the longitudinal field B_x:

$$\frac{\partial B_x}{\partial x}(x, \rho = 0) = -\frac{\mu_0}{4\pi} \frac{6\pi a^2 N I x}{(a^2 + x^2)^{5/2}} \tag{4.13}$$

By way of example, the values of the magnetic field and its longitudinal gradients are shown in table 4.1 for three different points on the TM with a DC current of 1 mA fed to the 2 400 turn LTP coil.

4.4 Force and torque amplitudes computation (Step 2)

Although the purpose of the magnetic experiment is *to determine* the remanent magnetic moment and the susceptibility of the TMs, a model implementation requires some nominal values to be used in the formulas of section 3.2.1. We make the assumption that the remanent magnetisation is *homogeneous* across the TMs, so that $\mathbf{M} = \mathbf{m}_0/V$, where $\mathbf{m}_0$ is the remanent magnetic moment and $V = (46\text{ mm})^3 = 10^{-4}\,\text{m}^3$. We adopt the following nominal values:

$$|\mathbf{M}| = \frac{|\mathbf{m}_0|}{V} = 2 \times 10^{-4}\ \text{A/m} \tag{4.14a}$$

$$\chi = 2.5 \times 10^{-5} \tag{4.14b}$$

The magnetic forces acting on the test mass include a term directly proportional to the gradient of the field, and another one proportional to the product of the field with its gradient —see equation (3.1). On the other hand, the torque expressions do not show any dependence on the product of the latter magnitudes —see equation (3.4). As a consequence, we can conclude that:

- Forces on the TMs are both linearly and quadratically dependent on the coil current.
- Torques on the TMs are only linearly dependent on the current.

We express the above in the following form:

$$\mathbf{F} = \mathbf{F}_1\, i + \mathbf{F}_2\, i^2 \qquad \mathbf{N} = \mathbf{N}_1\, i \tag{4.15}$$

where $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{N}_1$ are 3-component vectors in the usual physical space.

The scheme above will be used for forces and torques in time series calculations. Therefore, as a first step each of the vector values in the right hand sides of equations (4.15) will be computed. Thereafter, the appropriate time series will be generated.

4.4.1 Computation of the force (Steps 2.1 through 2.3)

The computation of the x-component of $\mathbf{F}_1$ and $\mathbf{F}_2$ is done as follows:

$$\begin{aligned} F_{1_x} &= \sum_{l_x=1}^{n_x}\sum_{l_y=1}^{n_y}\sum_{l_z=1}^{n_z}\left\{ M_x[l_x,l_y,l_z]\,\frac{\partial B_x}{\partial x}[l_x,l_y,l_z] + \right. \\ &+ M_y[l_x,l_y,l_z]\,\frac{\partial B_x}{\partial y}[l_x,l_y,l_z] + \\ &+ \left. M_z[l_x,l_y,l_z]\,\frac{\partial B_x}{\partial z}[l_x,l_y,l_z]\right\}\,\Delta x\Delta y\Delta z \end{aligned} \tag{4.16}$$

and

$$\begin{aligned} F_{2_x} &= \sum_{l_x=1}^{n_x}\sum_{l_y=1}^{n_y}\sum_{l_z=1}^{n_z}\left\{ \frac{\chi}{\mu_0} B_x[l_x,l_y,l_z]\,\frac{\partial B_x}{\partial x}[l_x,l_y,l_z] + \right. \\ &+ \frac{\chi}{\mu_0} B_y[l_x,l_y,l_z]\,\frac{\partial B_x}{\partial y}[l_x,l_y,l_z] + \\ &+ \left. \frac{\chi}{\mu_0} B_z[l_x,l_y,l_z]\,\frac{\partial B_x}{\partial z}[l_x,l_y,l_z]\right\}\,\Delta x\Delta y\Delta z \end{aligned} \tag{4.17}$$

Use of equation (4.2) gives $\Delta x\Delta y\Delta z/V = (n_x n_y n_z)^{-1}$. Using also a scalar product notation, the above formula can be cast in a cleaner form:

$$F_{1_x} = \frac{V}{n_x n_y n_z}\sum_{l_x=1}^{n_x}\sum_{l_y=1}^{n_y}\sum_{l_z=1}^{n_z}\left\{\mathbf{M}[l_x,l_y,l_z]\cdot\boldsymbol{\nabla} B_x[l_x,l_y,l_z]\right\} \tag{4.18}$$

and

$$F_{2_x} = \frac{V}{n_x n_y n_z}\sum_{l_x=1}^{n_x}\sum_{l_y=1}^{n_y}\sum_{l_z=1}^{n_z}\left\{\frac{\chi}{\mu_0}\,\mathbf{B}[l_x,l_y,l_z]\cdot\boldsymbol{\nabla} B_x[l_x,l_y,l_z]\right\} \tag{4.19}$$

The other components of the force are calculated in exactly the same fashion:

$$F_{1_y} = \frac{V}{n_x n_y n_z}\sum_{l_x=1}^{n_x}\sum_{l_y=1}^{n_y}\sum_{l_z=1}^{n_z}\left\{\mathbf{M}[l_x,l_y,l_z]\cdot\boldsymbol{\nabla} B_y[l_x,l_y,l_z]\right\} \tag{4.20}$$

and

$$F_{2_y} = \frac{V}{n_x n_y n_z} \sum_{l_x=1}^{n_x} \sum_{l_y=1}^{n_y} \sum_{l_z=1}^{n_z} \left\{ \frac{\chi}{\mu_0} \mathbf{B}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_y[l_x, l_y, l_z] \right\} \tag{4.21}$$

and, finally,

$$F_{1_z} = \frac{V}{n_x n_y n_z} \sum_{l_x=1}^{n_x} \sum_{l_y=1}^{n_y} \sum_{l_z=1}^{n_z} \{ \mathbf{M}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_z[l_x, l_y, l_z] \} \tag{4.22}$$

and

$$F_{2_z} = \frac{V}{n_x n_y n_z} \sum_{l_x=1}^{n_x} \sum_{l_y=1}^{n_y} \sum_{l_z=1}^{n_z} \left\{ \frac{\chi}{\mu_0} \mathbf{B}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_z[l_x, l_y, l_z] \right\} \tag{4.23}$$

4.4.2 Computation of the torque (Steps 2.4 through 2.6)

In order to compute the torques around the CoM of the TMs, we recall that the vector $\mathbf{r}$ in equation 3.4 originates in the CoM and points towards the chosen grid point. Therefore, in the following expressions vectors x, y and z will be defined as follows:

$$\begin{aligned} x[l_x] &= -L/2 + l_x \Delta x \ , \quad 1 \leq l_x \leq n_x \\ y[l_y] &= -L/2 + l_y \Delta y \ , \quad 1 \leq l_y \leq n_y \\ z[l_z] &= -L/2 + l_z \Delta z \ , \quad 1 \leq l_z \leq n_z \end{aligned} \tag{4.24}$$

where $[-L/2, -L/2, -L/2]$ is the position of the BLR (bottom-left-rear) corner with respect to the Centre of Mass of the TM. Therefore, the expression to compute the torque about the x-axis is:

$$\begin{aligned} N_{1_x} &= \frac{V}{n_x n_y n_z} \sum_{l_x=1}^{n_x} \sum_{l_y=1}^{n_y} \sum_{l_z=1}^{n_z} \Big\{ (M_y[l_x, l_y, l_z]\ B_z[l_x, l_y, l_z] - M_z[l_x, l_y, l_z]\ B_y[l_x, l_y, l_z]) + \\ &+ \ y[l_y]\ (\mathbf{M}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_z[l_x, l_y, l_z]) - z[l_z]\ (\mathbf{M}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_y[l_x, l_y, l_z]) \Big\} \end{aligned} \tag{4.25}$$

where the scalar product notation used in the previous section has also been used here in identical fashion. The other two components of the torque are evaluated likewise:

$$\begin{aligned} N_{1_y} &= \frac{V}{n_x n_y n_z} \sum_{l_x=1}^{n_x} \sum_{l_y=1}^{n_y} \sum_{l_z=1}^{n_z} \Big\{ (M_z[l_x, l_y, l_z]\ B_x[l_x, l_y, l_z] - M_x[l_x, l_y, l_z]\ B_z[l_x, l_y, l_z]) + \\ &+ \ z[l_z]\ (\mathbf{M}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_x[l_x, l_y, l_z]) - x[l_x]\ (\mathbf{M}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_z[l_x, l_y, l_z]) \Big\} \end{aligned} \tag{4.26}$$

and, finally,

$$\begin{aligned} N_{1_z} &= \frac{V}{n_x n_y n_z} \sum_{l_x=1}^{n_x} \sum_{l_y=1}^{n_y} \sum_{l_z=1}^{n_z} \Big\{ (M_x[l_x, l_y, l_z]\ B_y[l_x, l_y, l_z] - M_y[l_x, l_y, l_z]\ B_x[l_x, l_y, l_z]) + \\ &+ \ x[l_x]\ (\mathbf{M}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_y[l_x, l_y, l_z]) - y[l_y]\ (\mathbf{M}[l_x, l_y, l_z] \cdot \boldsymbol{\nabla} B_x[l_x, l_y, l_z]) \Big\} \end{aligned} \tag{4.27}$$

4.5 Current signals generation (Step 3)

4.5.1 Sinusoidal signals generation (Step 3.1)

In this step, a time series corresponding to a sinusoidal signal with a specific amplitude A, a specific frequency f and for a specific duration T is generated. The signal is then sampled at specific sampling rate, f_s. We thus have

$$i_{\rm noise\ free} = A\,\sin(2\pi f t) \longrightarrow i_{\rm noise\ free}[\kappa] = A\,\sin\left(2\pi\kappa\frac{f}{f_s}\right) \quad , \quad \kappa = 0, \ldots, N_s - 1 \tag{4.28}$$

where, clearly, $N_s = f_s T$. The sampling frequency is 10 Hz. T is the simulation time of the experiment and this should be the same global variable controlling this aspect in the whole LTP End-to-End simulator. Concerning the magnetic experiments, durations of about 10 000 seconds are sufficient for parameter estimation when a current 1 mA is injected into the coils. This duration is the one actually foreseen in flight, but it may vary depending on the requirements/availability constraints on the STOC model. It must be recalled that 10 000 seconds for the experiment duration in flight is a requirement for accurate estimation of the parameters χ and $\mathbf{M}$, while the purpose of the STOC model is to address effects which can be made apparent with somehow shorter integration times.

4.5.2 Current noise generation (Step 3.2)

Current noise can be generated out of the requirements specification document [RD4] or the DDS Algorithm user requirement document [RD10]. Here, although valid, a white noise time series with an amplitude density spectrum below $18\,\mathrm{mA}/\sqrt{\mathrm{Hz}}$ turns out to be a pessimistic approach.

To be more realistic, the current noise can be implemented out of the experimental spectrum measured during the hardware test performed in Barcelona and shown in [RD11]. Noise samples will thus be drawn directly from laboratory data, $n[\kappa]$ ($\kappa = 0, \ldots, N_s - 1$).

4.5.3 Addition of both time series (Step 3.3)

In this step both previous time series are added:

$$i[\kappa] = i_{noise\ free}[\kappa] + n[\kappa] \tag{4.29}$$

4.6 Force and torque time series (Step 4)

To obtain the corresponding force and torques time series, each of the amplitude values computed in the previous section, i.e., F_{1_x}, F_{2_x}, F_{1_y}, F_{2_y}, F_{1_z}, F_{2_z}, N_{1_x}, N_{1_y} and N_{1_z}, will be multiplied by $i[\kappa]/I$ or $i^2[\kappa]/I^2$, where I is the reference DC current (1 A) used in the computation of the magnetic induction field in section 4.3.

Explicitly, for the components of the force and torque the scheme is

$$\mathbf{F}(t) = \frac{i(t)}{I}\,\mathbf{F}_1 + \left(\frac{i(t)}{I}\right)^2 \mathbf{F}_2 \quad \longrightarrow \quad \mathbf{F}[\kappa] = \frac{i[\kappa]}{I}\,\mathbf{F}_1 + \left(\frac{i[\kappa]}{I}\right)^2 \mathbf{F}_2 \tag{4.30}$$

and the same reasoning for the torque components:

$$\mathbf{N}[\kappa] = \frac{i[\kappa]}{I}\,\mathbf{N}_1 \tag{4.31}$$